## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 5 Investigation (Core)
May/June 2021
1 hour 10 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 36 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 12 pages. Any blank pages are indicated.

Answer all the questions.

## INVESTIGATION

NEAREST NEIGHBOURS
This investigation looks at the distances from the origin $(0,0)$ to the centres of squares and hexagons that form a pattern.

The diagram shows a pattern of congruent squares.
A dot marks the centre of each square.
The coordinates of some of the centres are $(0,0),(-3,-2)$ and $(3,1)$.


1 (a) Complete this table, using a tick in each row, to show whether a point is

- at the centre of a square
- inside a square, but not at its centre
- on the side of two squares
- where four squares meet.

The first three have been done for you.

| Point | At the centre of <br> a square | Inside a square, <br> not at its centre | On the side of <br> two squares | Where four <br> squares meet |
| :---: | :---: | :---: | :---: | :---: |
| $(0,0)$ | $\checkmark$ |  |  |  |
| $(-1,0.5)$ |  |  | $\checkmark$ |  |
| $(0.5,0.5)$ |  |  |  | $\checkmark$ |
| $(1.5,1.5)$ |  |  |  |  |
| $(0,1.5)$ |  |  |  |  |
| $(0.25,0)$ |  |  |  |  |
| $(100.5,99.5)$ |  |  |  |  |

(b) Give a reason for your answer for (100.5, 99.5).
$\qquad$
$\qquad$

2 Each dot on the grid marks the centre of a square.


The nearest dots to $(0,0)$ are $(-1,0),(0,1),(1,0)$ and $(0,-1)$.
These four dots are the 1st nearest neighbours.
The 2nd nearest neighbours to $(0,0)$ are $(1,1),(1,-1),(-1,-1)$ and $(-1,1)$.
All nearest neighbours have integer coordinates.
(a) One of the 3 rd nearest neighbours to $(0,0)$ is $(2,0)$.

Find the other 3rd nearest neighbours and write down their coordinates.
$\qquad$
(b) Find the coordinates of all the 4th nearest neighbours to $(0,0)$.
$\qquad$
$\qquad$

3 You can find the distance, $d$, from $(0,0)$ to the point $(a, b)$ using Pythagoras' Theorem.

$$
d^{2}=a^{2}+b^{2}
$$

(a) Show that the distance of a 4th nearest neighbour from $(0,0)$ is $\sqrt{5}$.
(b) Here are four points and their coordinates.
$A(20,5) \quad B(7,24) \quad C(-7,24) \quad D(0,25)$

Which of these points are a distance of 25 units from $(0,0)$ ?
(c) There are more than 10 nearest neighbours to $(0,0)$ with $d=5$.

Four of them are $(0,5),(5,0),(-5,0)$ and $(0,-5)$.
On the grid, mark with a cross all the nearest neighbours to $(0,0)$ with $d=5$.


4 Here is a pattern of congruent regular hexagons.
Each dot marks the centre of a hexagon.
The triangle shown is an equilateral triangle.

(a) (i) Explain why the triangle shown is an equilateral triangle.
$\qquad$
The dots at the centre of the hexagons have coordinates as shown on the grid below.

(ii) A line is made by joining the points $(3,0)$ and $(4,1)$.

Work out the size of the acute angle between this line and the $x$-axis.
(b) The point at the centre of each hexagon has six 1st nearest neighbours.
(i) Complete this list of the 1st nearest neighbours to the point $(1,1)$.

$$
\begin{equation*}
(0,1), \quad(0,2),(1,2), \tag{2}
\end{equation*}
$$

(ii) Find, in terms of $a$ and $b$, the coordinates of the six 1 st nearest neighbours to the point $(a, b)$.
$\qquad$
(c) A student suggests that an estimate of $d$, the distance from $(0,0)$ to the point $(a, b)$, is $d=\sqrt{a^{2}+b^{2}+a b}$.

The distance between any point and its 1 st nearest neighbours on this grid is 1 cm .


Does the student's formula give a good estimate for the distance from $(0,0)$ to $(4,-3)$ ? Show how you decide.

5 The circles show the 1st, 2nd, 3rd, and 4th nearest neighbours to the point $\mathbf{X}$.

(a) Complete this table.

| Nearest neighbour | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of nearest neighbours |  |  |  |  |  | 6 | 12 | 6 |

(b) A computer calculates that there is a total of 9 points for a certain nearest neighbour distance.

Explain why the computer is probably wrong.

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